Crossed modules and H^3

February-17-14 3:06 PM

(140210) A crossed module (e.g. Martins' arXiv:0801.3921) models $\partial\colon \pi_2(X,A) \to \pi_1(A)$: a group homomorphism $\partial\colon E \to G$ with an action $G \circlearrowleft E$ s.t. (1) $\partial(ge) = g(\partial e)g^{-1}$, (2) $(\partial e)f = efe^{-1}$ (contains the "class in $H^3(\pi_1,\pi_2)$" information when $\cancel{X}$ A $= X^1$).

$$\pi_2(A) \to \pi_2(X) \longrightarrow \underset{E}{\pi_2(X,A)} \xrightarrow{\partial} \underset{G}{\pi_1(A)} \to \pi_1(X) \to$$

If $A = X^1$, $\pi_1(X) = \pi_1(A)/\pi_2(X,A)$ and $\pi_2(A) = 0$.
(and E is Abelian)

So let $\pi\colon E \to G$ be a crossed module with Abelian E.

Think $\pi_1 X = G/\partial E$ & $\pi_2 X = \ker \partial$

<u>Question</u> Can I construct a class in $H^3(G/\partial E, \ker\partial)$?

Meaning, $\varphi\colon (G/\partial E)^3 \longrightarrow \ker\partial$ s.t. $d\varphi = 0$?

Seems impossible.